An Image Processing Approach to Pre-compensation for Higher-Order Aberrations in the Eye

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ABSTRACT

Human beings rely heavily on vision for almost all of the tasks that are required in daily life. Because of this dependence on vision, humans with visual limitations, caused by genetic inheritance, disease, or age, will have difficulty in completing many of the tasks required from them. Some individuals with severe visual impairments, known as high-order aberrations, may have difficulty in interacting with computers, even when using traditional means of visual correction (e.g., spectacles, contact lenses). This is, in part, because these correction mechanisms can only compensate for the most regular (low-order) distortions or aberrations of the image in the eye. This paper proposes an image processing approach that will pre-compensate the images displayed on the computer screen, so as to counter the effect of the eye’s aberrations on the image. The characterization of the eye required to perform this customized pre-compensation is the eye’s Point Spread Function (PSF), which can now be measured by a new generation of ophthalmic instruments generically called “Wavefront Analyzers.” The characterization provided by these instruments also includes the “higher-order aberration components” and could, therefore, lead to a more comprehensive vision correction than traditional mechanisms. The methods presented here will be explained in terms of their theoretical foundation and illustrated with results from the correction of aberrations introduced by a lens with known and constant PSF.

Keywords: Point spread function, Optical transfer function, Retina, Deconvolution, Pre-deblurring, Deblurring

1. INTRODUCTION

Visual perception is the primary source of information about the surrounding environment that humans have. Through the evolutionary process, humans have developed a very refined sense of vision, seeing in three dimensions with high resolution and color [1]. The sense of sight is so important that, when limited by age or disease, it reduces a person’s ability to perform otherwise ordinary tasks, such as interacting with a computer system.

Visual perception of objects in the physical world is determined by the formation of their images on the retina, located inside the human eye [2]. However, the natural visual system of some individuals does not accurately map images from the outside world onto their retinas. The most common of these visual impairments include myopia, hyperopia, and astigmatism. All of them result in a retinal representation of a point of light that is not confined to a single point on the retina. The distortion in the mapping of external images onto the retina is represented by the eye’s “wavefront aberration function.” Myopia, hyperopia, and astigmatism are referred to as low-order aberrations because the distortion they introduce can be modeled by first or second order Zernike polynomials. Currently, spectacles or contact lenses correct these low-order impairments by modifying external images before they reach the eye.

There are cases, however, in which the low-order Zernike model is not sufficient to describe the aberration of the eye. With the recent advances in Wavefront Sensing technology, it is now feasible to accurately model the high-order aberrations present in each person’s eye and thus obtain good models of various aberrations currently not correctable through conventional means (e.g., spectacles, contact lenses). Using these models, it would be possible to provide persons with currently uncorrectable aberrations a new alternative to enhance their interaction with computers.

In contrast with the optical correction of visual limitations, the approach described here is based on modifying the image at its source, i.e., in applying image processing modifications on the image to be displayed on-screen before it is shown to the user, based on the knowledge of his/her wavefront aberration function. The aim of the pre-compensation proposed is to modify the intended display image in a way that is opposite to the effect of the wavefront aberration of the eye. Once this is achieved, the result is displayed to the viewer so that the effect of the wavefront aberration in the viewer’s eye will “cancel” the pre-compensation resulting in the projection
of an undistorted version of the intended image on the retina.

2. METHODOLOGY

Imaging in the eye as a convolution process

A recent shift in the interpretation of optics has been in the characterization of optical systems, including the human eye, as linear, shift invariant systems described by their point spread function (PSF) [2]. Therefore, the eye can be characterized by its PSF, \( T(x, y) \), and the retinal image, \( R(x, y) \), can be found by convolving (denoted by \( \otimes \) ) the input to the system, i.e., the image of the object to be seen, \( I(x, y) \), with the PSF of the eye:

\[
R(x, y) = I(x, y) \otimes T(x, y)
\]  

(1)

When viewing a computer screen, the real world image, \( I(x, y) \), is stored as a digital image, \( DI(x, y) \), which will be displayed to the user. The retinal image, in this case, will be formed by convolution of the on-screen image and the PSF of the viewer’s eye:

\[
R(x, y) = DI(x, y) \otimes T(x, y)
\]  

(2)

In this case, however, the on-screen image to be displayed to the user can be manipulated in advance. If the inverse PSF of the viewer’s eye, \( T^{-1}(x, y) \), could be defined, then an image, \( RD(x,y) \), which is the result of convolving the intended digital image, \( DI(x, y) \), with this inverse function could be shown to the viewer:

\[
RD(x, y) = DI(x, y) \otimes T^{-1}(x, y)
\]  

(3)

Under these circumstances, substituting the ordinary \( DI(x,y) \) with the pre-compensated \( RD(x,y) \), in (2):

\[
R(x, y) = \{DI(x, y) \otimes T^{-1}(x, y)\} \otimes T(x, y) = DI(x, y)
\]  

(4)

This means that the user would perceive in \( R(x,y) \) an undistorted version of the intended digital image, \( DI(x,y) \). The image represented by the braces in (4) can also be considered to be the result of deconvolving the PSF, \( T(x,y) \), from the intended image, \( DI(x,y) \). This is the pre-compensated image to be displayed to the viewer. In practice, the deconvolution process may be performed more efficiently in the frequency domain, as discussed below. In any case, it is clear that, in order to obtain \( RD(x, y) \), the PSF of the eye must be known or estimated.

Evaluation of the PSF of the human eye

The PSF of the human eye can be found indirectly through what is known as the wavefront aberration function, \( W(x,y) \). This function represents the deviation of the light wavefront from a purely spherical pattern as it passes the pupil on its way to the retina [3]. In an unaberrated eye, the refracted light is organized in the form of a uniform spherical wavefront, converging to the paraxial focal point on the retina.

Recently developed “Wavefront Analyzers” based on the Hartmann-Shack Principle have made it possible to measure the wavefront aberration function for the human eye. The wavefront aberration function, \( W(x,y) \), is the primary component of the pupil function, \( P(x,y) \). The pupil function incorporates the complete information about the imaging properties of the optical system [4], and is defined as:

\[
P(x, y) = A(x, y)e^{-j2\pi n W(x,y)}
\]  

(5)

where \( A(x,y) \) is the amplitude function describing the relative efficiency of light passing through the pupil (usually given a value of one), and \( n \) is the index of refraction [3].

According to the Fourier optics relationships in the eye, knowledge of the pupil function can be used to determine the optical transfer function, OTF, which is “one of the most powerful descriptors of imaging performance for an optical system” [3]. The OTF is a complex function whose magnitude is the modulation transfer function (MTF) and whose phase is the phase transfer function (PTF) [3].

The OTF can be found by convolving the pupil function, \( P(x,y) \), with its complex conjugate, \( P^*(-x,-y) \) [2],[3]:

\[
O(fx, fy) = P(x, y) \otimes P^*(-x,-y)
\]  

(6)

which is equivalent of saying that the OTF is the autocorrelation of the pupil function. The PSF and the OTF are a Fourier transform pair [5]:

\[
O(fx, fy) = F\{T(x, y)\}
\]  

(7)

The wavefront aberration function, \( W(x,y) \), is usually represented by a Zernike polynomial expansion. These polynomials are two-dimensional functions that form a complete orthogonal basis set defined on the unit disc. They have been used in optical engineering for over 60 years, [6], [7]. Recently, the Zernike polynomials have been applied to the characterization of aberrations of the human eye [8], [9], [10]. Modern ophthalmic wavefront analyzers provide an approximation to the wavefront aberration function measured from the subject as a combination of Zernike polynomials. As indicated by equations (5), (6), and (7), this characterization of the optics of the eye can be used to determine its PSF and OTF, thus enabling the implementation of the deconvolution concept expressed in equations (3) and (4), as detailed below.
Inverse Filtering as a means of deconvolution

Inverse filtering is traditionally used in image processing to restore an image \( U(x,y) \) from a degraded image \( G(x,y) \), assuming a known degradation function, \( H(x,y) \). As stated above, the input and output of any linear system are related through the convolution operator. That is,

\[
G(x, y) = U(x, y) \otimes H(x, y) \tag{8}
\]

If the Fourier principles of convolution are applied, \( U(x,y) \) can be obtained as follows:

\[
U(x, y) = F^{-1}\left\{ \frac{F\{G(x, y)\}}{F\{H(x, y)\}} \right\} \tag{9}
\]

where \( F\{ \} \) and \( F^{-1}\{ \} \) denote the Fourier transform and the inverse Fourier transform, respectively.

In the context of pre-compensation of a digital image to be shown to the viewer, the objective is to deconvolve the PSF of the viewer’s eye, \( T(x, y) \), from the intended digital image, \( DI(x, y) \), in order to derive the pre-compensated display image to show on-screen, \( RD(x, y) \). Making the corresponding substitutions in equation (9), the calculation of \( RD(x, y) \) is practically accomplished as:

\[
RD(x, y) = F^{-1}\left\{ \frac{F\{DI(x, y)\}}{F\{T(x, y)\}} \right\} \tag{10}
\]

This process is commonly known as inverse filtering in frequency, and is equivalent to two-dimensional deconvolution in the spatial domain. It should be noted that, according to equation (7), the denominator within the braces of equation (10) is the OTF of the viewer’s eye.

But this implementation of inverse filtering has several limitations, especially for values of the PSF or its frequency counterpart, the OTF, which are close to zero. A common approach to circumvent this problem is the use of Minimum Mean Square Error filtering [11]. In the context of the problem at hand, this approach obtains the pre-compensated image as:

\[
RD(fx,fy) = \frac{1}{\left| T(fx,fy) \right|^2 + K} T(fx,fy) DI(fx,fy) \tag{11}
\]

where \( RD(fx,fy) \), \( T(fx,fy) \), and \( DI(fx,fy) \) are the Fourier transforms of \( RD(x,y) \), \( T(x,y) \), and \( DI(x,y) \), respectively. In addition to the limitation of values in the pre-compensated image, the approach indicated in equation (11) helps reduce the impact of modeling or measurement noise inherent to the definition of the eye’s PSF. The \( K \) factor in equation (11) should be proportional to the magnitude of the estimation noise in the PSF approximation.

3. RESULTS AND DISCUSSION

Testing

Quantification of the results was done with the participation of human subjects. In order to proceed with the testing of the pre-compensation method proposed, human subjects having a satisfactory standard visual acuity of 20/20 [13], either corrected or natural, were recruited. Then, a refractive error, specifically a defocus, was artificially introduced in their field of view by means of a pre-selected lens. A similar procedure was used by Sonksen to test visual function [14].

In order to provide a well-known and standardized method of reporting visual performance, a standard Eye Test Chart was used. The characters of a Bailey-Lovie visual acuity chart with ‘Sloan’ letters (Figure 1) were reproduced as properly sized digital images for the tests.

![Figure 1- Bailey-Lovie Eye Test Chart](image_url)

Simulated Experiment for -6.00 Diopter Sphere Lens

The PSF of a -6.0 diopter (D) lens was obtained by means of equation (11) using the image of a point source of light in dark background, which replaces \( T(x,y) \), and the image of the same scene, viewed through the lens replacing \( DI(x,y) \). This PSF estimation, as well as the generation of the pre-compensated image, was accomplished using MATLAB®. Figure 2 shows the analytical PSF obtained, using the wavefront aberration function corresponding to a -6.0 D lens, scaled and resized. A comparison of the two PSF, both analytical and empirical is necessary to ensure the proper pre-compensation. Once the analytical PSF is verified to be similar to the empirical PSF, the analytical is used to
apply the pre-compensation. The empirical PSF contains a
great deal of noise which hinders the deconvolution
process, making even more ill-posed.

Additionally, the analytical PSF has been cropped and
padded to match the image size of a line from a standard
eye test chart, shown in Figure 3. This is necessary due to
the fact that both the image and the PSF need to be of the
same size in order to implement the minimum mean
square error deconvolution described by equation (11).
Another key factor to this process is that the eye test chart
lines, when displayed on-screen, must be equivalent in
size to an actual eye test chart. This is important because
it ensures that the test can be properly administered.

Figure 4 shows a simulation of the upper line of the eye
test chart, as seen by the experimental subjects through
the –6.0 D lens. This simulation is obtained by
convolution of the analytical PSF (Figure 2) with the
image in Figure 3.

Figure 5 shows the result of applying the pre-
compensation process to the image in Figure 3. This is the
image that compensates for the aberration introduced by
the -6.0 D lens. This image was produced using equation
(11) with a K value of 0.0005.

Figure 6 shows a simulation of the result of the pre-
compensation process obtained by convolving Figure 5
with the PSF of the lens (Figure 2).

**Human Subject Testing Results**

The results of testing with human subjects are shown in
Figure 7. This figure shows the visual acuity in logMAR
units for fourteen subjects. Each of their eyes was studied
independently resulting in twenty-eight visual acuity
scores. With respect to Figure 7, the bottom trace
indicates the visual acuities obtained for the eye test
without any blur or pre-compensation. The top trace
(dashed line) indicates the values through the blur, and the
center trace represents the values obtained from the pre-
compensated eye test. A smaller value of visual acuity
indicates the ability of the subject to correctly read further
down the chart in a standard eye test. For reference, a
logMAR value of 1.4 at a viewing distance of two meters
represents a misreading of all of the largest letters on the
eye test chart.

As can be seen from Figure 7, the pre-compensation
partially restored the vision of the subjects, allowing them
to read further down the eye test chart.

![Figure 2- Analytical PSF of the –6.0 D lens used in the tests.](image)

![Figure 3- Digital Representation of one line from a standard eye test chart](image)

![Figure 4- Simulated blur caused by the PSF in Figure 1](image)

![Figure 5- Resulting pre-compensated image](image)

![Figure 6- Simulated image of what the subject would see when viewing Figure 3 through the -6.0 diopter lens.](image)

![Figure 7- Results with human subjects](image)
4. CONCLUSION

This paper has proposed a framework for pre-compensation of digital images before they are displayed to low-vision users on a computer screen in order to compensate for their visual limitations. The results shown in Figure 7 demonstrate that this is indeed possible, and that the method presented in this paper, when applied to digital on-screen images, partially restores the vision of subjects with an artificial, second-order aberration introduced into their field of view. Although this example pre-compensates for a low-order aberration, it should be noted that this framework also contemplates the correction of higher-order optical aberrations that are not addressed by current visual correction methods.

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5. REFERENCES